

# REPORT DOCUMENTATION PAGE

*Form Approved  
OMB NO. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimates or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	August, 1996	<i>Final 15 Jun 92 - 14 Jun 96</i>
4. TITLE AND SUBTITLE		5. FUNDING NUMBERS
Hp-ADAPTIVE FINITE ELEMENT METHODS FOR TIME DEPENDENT PROBLEMS WITH APPLICATIONS TO STRESS WAVES IN SOLIDS		<i>DAAL 03-92-G-0253</i>
6. AUTHOR(S)		
J. Tinsley Oden		
7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)		8. PERFORMING ORGANIZATION REPORT NUMBER
The University of Texas at Austin TICAM, TAY 2.400 Austin, TX 78712		
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSORING / MONITORING AGENCY REPORT NUMBER
U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211		<i>ARO 30297.9-MA</i>
11. SUPPLEMENTARY NOTES		
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.		
12a. DISTRIBUTION / AVAILABILITY STATEMENT		12 b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.		

## 13. ABSTRACT (Maximum 200 words)

The general goals of this project included the development of new, high-order, adaptive methods for the computer simulation of stress-wave propagation phenomena in solid mechanics, particularly elastodynamics.

The motivation for using adaptive methodologies is to control and optimize the computational process, to use a posteriori error estimates to optimize meshes and spectral orders of approximation. Such orchestrated meshing can produce exponential rates of convergence, thereby allowing complex simulations to be done using orders-of-magnitude fewer unknowns than standard methods.

19961023 232

DTIC QUALITY INSPECTED 1

14. SUBJECT TERMS		15. NUMBER OF PAGES
A posteriori error estimates, adaptivity.		3
		16. PRICE CODE
17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED
		20. LIMITATION OF ABSTRACT
		UL

**H<sub>p</sub>-ADAPTIVE FINITE ELEMENT METHODS FOR TIME  
DEPENDENT PROBLEMS WITH APPLICATIONS TO  
STRESS WAVES IN SOLIDS**

**FINAL PROGRESS REPORT**

Professor J. Tinsley Oden  
The University of Texas at Austin

August, 1996

**U.S. ARMY RESEARCH OFFICE**

**DAAL03-92-G-0253**

**THE UNIVERSITY OF TEXAS AT AUSTIN**

**APPROVED FOR PUBLIC RELEASE;  
DISTRIBUTION UNLIMITED.**

THE VIEWS, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE  
THOSE OF THE AUTHOR(S) AND SHOULD NOT BE CONSTRUED AS AN  
OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY, OR DECISION,  
UNLESS SO DESIGNATED BY OTHER DOCUMENTATION.

**FINAL PROGRESS REPORT  
DAAL03-92-G-0253**

***Statement of Problem Studied***

The general goals of this project included the development of new, high-order, adaptive methods for the computer simulation of stress-wave propagation phenomena in solid mechanics, particularly elastodynamics.

The motivation for using adaptive methodologies is to control and optimize the computational process, to use a posteriori error estimates to optimize meshes and spectral orders of approximation. Such orchestrated meshing can produce exponential rates of convergence, thereby allowing complex simulations to be done using orders-of-magnitude fewer unknowns than standard methods.

***Summary of Results***

This project has led to a number of completely new and powerful methods for the computer simulation of complex problems, including, in particular, problems of wave propagation. These include the following:

- High-Order Multistage-Taylor Galerkin Methods. These represent one of the only new unconditionally stable high-order time integration schemes developed in decades. They are designed to overcome a well-documented deficiency of splitting methods: the loss of time accuracy due to splitting of boundary conditions.
- A Posteriori Error Estimation Methods. New techniques for the estimation of error in numerical approximations of wave problems in two space dimensions have been developed; theorems have been established to guarantee that rigorous error bounds are possible.
- Adaptive methods. Local error estimates provide data for adaptively changing mesh sizes and spectral orders to optimize hp meshes and accelerate convergence.
- Parallel Discontinuous Methods. A surprisingly parallelizable scheme based on Discontinuous Galerkin methods has been developed and tested on model problems.
- Clouds: A New family of Meshless Methods. Error estimates and preliminary results on the mathematical foundations of a new type of meshless technique for solving partial differential equations have been established.

***List of Publications***

Safjan, A. and Oden, J.T., "High Order Taylor-Galerkin and Adaptive hp Methods for Second-Order Hyperbolic Systems: Application to

- Elastodynamics," *Computer Methods in Applied Mechanics and Engineering*, v. 103, nos. 102, pp. 187-230.
- Ainsworth, M. and Oden, J.T., "A Procedure for A Posteriori Error Estimation for hp-Finite Element Methods," *Computer Methods in Applied Mechanics and Engineering*, Vol. 101, 1992, pp. 73-96.
- Ainsworth, M., Oden, J.T., and Wu, W., "A Posteriori Error Estimation for hp-Approximations in Elastostatics," *J. Appl. Num. Math.*, 14, 1994, pp. 23-54.
- Oden, J.T., "Optimal hp-Finite Element Methods," TICOM Report 92-09
- Oden, J.T., "Optimal hp-Finite Element Methods," *Computer Methods in Applied Mechanics and Engineering*, vol. 112, 1994, pp. 309-351.
- Safjan, A. and Oden, J.T., "High-Order Taylor-Galerkin and Adaptive hp-Methods for Hyperbolic Systems," *J. Computational Physics*, vo. 120, 1995, pp. 206-230.
- Oden, J.T., Safjan, A., Geng, P. and Demkowicz, L., "High-Order, Multi-Level Adaptive Time-Domain Methods for Structural Acoustics Simulations," *Large-Scale Structures in Acoustics and Electromagnetics*, National Academy of Sciences, September 1994, TICAM Report 94-14.
- Duarte, C. Armando and Oden, J.T., "H-p Clouds -- an h-p Meshless Methods," *Numerical Methods for Partial Differential Equations*, Vol. (1996), (in press).
- Duarte, C. Armando and Oden, J.T., "A New Meshless Method to Solve Boundary-Value Problems," Proceedings of the XVI CILAMCE-Iberian Latin American Conference on Computational Methods for Engineering, Curitiba, Brazil, November, Machado, R.D. (ed)., 1995, pp. 90-95.
- Duarte, C. Armando and Oden, J.T., "Hp Clouds -- A Meshless Method to Solve Boundary-Value Problems," TICAM Report 95-05, May 1995.
- Duarte, C. Armando and Oden, J.T., "An h-p Adaptive Method Using Clouds," TICAM Report 96-07, February 1996.

### *Participating Scientific Personnel*

- J. Tinsley Oden, Principal Investigator  
A. Safjan, Ph.D. received in Fall 1993  
T. Zohdi, Graduate Student  
C. Armando Duarte, Graduate Student  
K. Vemaganti, Graduate Student